الامتحان مكون من (4) أسئلة مكتوبة فى صفحة واحدة و المطلوب الإجابة على كل الأسئلة.

Time: 3 Hours

(1)(a)Test the series: (i) $\sum\limits_{n=1}^{\infty} \dfrac{2^n}{n + 4^n}$ (ii) $\sum\limits_{n=1}^{\infty} \dfrac{(-1)^n}{(2n+1)^{2n}}$

  (b)Determine the interval of convergence of the series: $\sum\limits_{n=1}^{\infty} \dfrac{n}{3^n}(2x+1)^n$

  (c)Find the first partial derivatives of the function: $f = x^2 e^{2z} + \ln(y^2 + 3z)$

(2)(a)Find the tangent plane to the surface $4x^2 - y^2 + 3z^2 = 10$ at $(2,- 3, 1)$

  (b)Find the extrema of the function: $f(x,y) = x^3 - y^3 + 3xy$

  (c) Find the extrema of the function: $f(x,y) = xy$ subject to $4x^2 + y^2 = 8$

(3)(a)If $f = x^2 y^3 + e^{2x} \sin z$. Find ∇f at the point $(1, 2, 0)$

  (b)If $\overline{U} = (xy + 2z)i + (y + \cos z)j + \ln(2x + 3z)k$. Find $\nabla.\overline{U}$, $\nabla x \overline{U}$

  (c)Find the curvature and the circle of curvature of the curve $y = x^3 - 2x + 2$
    at the point $(1,1)$

(4)Solve the following differential equations:

  (a)$(1 + 2ye^{2x})dx + (e^{2x} + \sin y)dy = 0$ (b) $y` - \dfrac{2}{x}y = x^3 + x$

  (c) $y`` - 4y` + 3y = 6 + 4e^x$      (d) $y`` + 4y = 4x + 5\cos 3x$

  (e)$(x^2 D^2 - 3xD + 4)y = x^3 + 2$ (f) $y`` - 4y` + 4y = \dfrac{1}{x}e^{2x}$

Good Luck   Dr. Mohamed H. Eid